The additional condition of zero heat flux at $x = L$ is thus

$$\lambda_s T'(L^-) + \lambda_g t'(L^-) = 0 \tag{2}$$

provided as above we neglect radiation from the inner surface. If λ_g is set equal to zero, then Eq. (2) leads to the Mills-Landis result, $T'(L^-) = 0, t \equiv T \equiv t_0 = t_c$; consideration of $\lambda_g \neq 0$ relieves us of that constraint.

Before proceeding with our analysis we wish to follow two suggestions of D. R. Kassoy regarding this reply. He suggested that we be explicit concerning 1) the role of Eq. (1) in our strategy and 2) the conductive heat flux q_c. We consider it convenient in carrying out the analysis to assume that in Eq. (1), $\rho v c_p$, t_0, and t_c are specified and thus that a priori only the difference, $q_r - q_c$, is specified. With respect to q_c we note that $q_c = -\lambda_g t'(0^-)$. If we were considering a strictly one-dimensional problem, corresponding to a doubly infinite domain, $x \to \pm\infty$, then the correct boundary conditions would be altered, q_c would not appear explicitly, but $\lambda_g t'(0^-)$ would. However, we are treating as one-dimensional only the semi-infinite region $x \geq 0$; we do so since we consider this a model for the case of practical interest involving a multidimensional, high-speed boundary layer flowing on the external surface $x = 0^-$. In this case we are justified in assuming that q_c is given and that $T(0) = t(0) = t_0$ for both suction and injection.

With Eq. (2) considered we outline our corrected analysis: Eq. (3) of Ref. 1 becomes

$$\rho v c_p t' - \lambda_g t'' = h(T - t) = \lambda_s T'' \tag{3}$$

subject to the conditions of Eq. (4) of Ref. 1 and to Eq. (2) above. An integration of the extremes of Eq. (3), and imposition of the conditions at $x = L[t = t_c, \lambda_s T'(L^-) + \lambda_g t'(L^-) = 0]$ leads to the equation

$$\rho v c_p (t - t_c) = \lambda_g t'(x) + \lambda_s T'(x) \tag{4}$$

One consequence of Eq. (4) is that at $x = 0$, where $t = t_0$, and where the right hand side equals $q_r - q_c$ we recover Eq. (1).

The analysis continues as follows: Differentiation of the first two parts of Eq. (3), elimination of T' by Eq. (4), and nondimensionalization as in Ref. 1 with the added parameter $\epsilon \equiv (\lambda_g/\lambda_s)$ results in a single equation for the distribution of the gas temperature, namely

$$\alpha\epsilon\hat{t}'''(\xi) - \hat{t}'' - \alpha\beta(1 + \epsilon)\hat{t}' + \beta\hat{t} = \beta\hat{t}_c \tag{5}$$

which is to be solved subject to the conditions

$$\hat{t}(0) = 1, \hat{t}(1) = \hat{t}_c, \hat{t}'(0) = \alpha\epsilon\hat{t}''(0)$$

The latter derives from Eq. (3) and from the requirement that $\hat{T}(0) = \hat{t}(0) = 1$. After Eq. (5) is solved, the temperature distribution within the solid is obtained by algebra from Eq. (3) as

$$\hat{T} = \hat{t} + (\alpha\beta)^{-1}\hat{t}' - (\epsilon/\beta)\hat{t}'' \tag{6}$$

The two conditions on $\hat{T}(\xi)$ $\hat{T}(0) = 1, \hat{T}'(1) = -\epsilon\hat{t}'(1)$ have been incorporated in the solution for $\hat{t}(\xi)$.

For the practically interesting case of $\alpha\epsilon \ll 1$ all the roots of the characteristic equation are real; in this case the solution of Eqs. (5) and (6) may be written as

$$(\hat{t} - \hat{t}_c)/(1 - \hat{t}_c) = A_1 e^{\lambda_1\xi} + A_2 e^{\lambda_2\xi} + A_3 e^{\lambda_3}$$

$$(\hat{T} - \hat{t}_c)/(1 - \hat{t}_c) = A_1 e^{\lambda_1\xi}[1 + (\alpha\beta)^{-1}\lambda_1 - (\epsilon/\beta)\lambda_1^2] + A_2 e^{\lambda_2\xi}[1 + (\alpha\beta)^{-1}\lambda_2 - (\epsilon/\beta)\lambda_2^2] + A_3 e^{\lambda_3\xi}[1 + (\alpha\beta)^{-1}\lambda_3 - (\epsilon/\beta)\lambda_3^2]$$

where the λ_i's are the roots of the characteristic equation

$$\alpha\epsilon\lambda^3 - \lambda^2 - \alpha\beta(1 + \epsilon)\lambda + \beta = 0$$

and where the A_i's are the arbitrary constants determined by

$$\begin{vmatrix} 1 & 1 & 1 \\ e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} \\ \lambda_1(1 - \epsilon\alpha\lambda_1) & \lambda_2(1 - \epsilon\alpha\lambda_2) & \lambda_3(1 - \epsilon\alpha\lambda_3) \end{vmatrix} \begin{vmatrix} A_1 \\ A_2 \\ A_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

Perhaps sufficient for our response to the criticism of Mills and Landis and more interesting than results from numerical analysis of this exact solution is the approximate solution which results from letting $\epsilon \to 0$. Some rough calculations based on the exact solution suggest the existence of an inner layer near $\xi = 1$ with an outer solution $\hat{t} \equiv \hat{T} \equiv 1$. Thus we introduce a new inner variable $\tilde{\xi} = (1 - \xi)\epsilon^{-1} > 0$, and find from Eq. (5) that the first order inner solution for $\hat{t}$ is

$$(1 - \hat{t})/(1 - \hat{t}_c) = e^{-\tilde{\xi}/\alpha} = e^{-(1-\xi)/\epsilon\alpha}$$

The first-order inner solution for $\hat{T}$ is found to be $\hat{T} \equiv 1$.

In conclusion we express our appreciation to Mills and Landis for exposing the error in our previous analysis and for forcing us to carry out the correct calculation.

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Comments on "Onset of Surface Combustion in Still Atmosphere"

CYE H. WALDMAN*
Office National d'Etudes et de Recherches Aérospatiales, Châtillon, France

IN Ref. 1 Liu has considered the problem of combustion initiation under surface reaction conditions in connection with fire prevention in space capsule interiors. The purpose of this Comment is twofold. First, to demonstrate several errors in Liu's analysis, then to show that related studies, of solid-propellant ignition, have already solved the problem posed in Ref. 1, and more thoroughly.

The governing equations describing the transient behavior of an oxidant containing gas adjacent to a combustible solid surface are the continuity equation, the momentum equation, the species conservation equations, and the heat conduction (or energy) equations for both the solid and gas phases. The momentum equation can generally be neglected in the absence of forced convection† owing to a constant gas phase pressure while the continuity equation can be absorbed or decoupled by appropriate transformation. In the case of surface combustion it is sufficient to consider only the oxidizer diffusion equation among the species conservation equations. Of the remaining equations, Liu[1] has neglected the heat conduction in the solid. This is a serious omission, however, because it has been demonstrated previously,[2] for a similar problem, that solid phase heat transfer can be several orders of magnitude greater than gas phase heat transfer (principally due to the much greater density of the solid). This justifies

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* Postdoctoral Fellow of the National Science Foundation, Washington, D.C. Member AIAA.

† Forced convection is taken here to mean that part of the flow which is not associated with mass injection or moving boundaries.

neglecting the gas phase energy equation, not the solid phase energy equation as done in Ref. 1. In addition, the attempt to account for solid phase heat transfer in Eq. (14) of Ref. 1 is inadequate because of the coupled nature of the equations.

Next we consider Liu's treatment of the equations. After transforming by a similarity variable there results Eq. (10) of Ref. 1. In this equation the time derivative has disappeared, thus losing part, but not all of the transient behavior. That the time derivative should be retained is evident from the fact that the Damköhler number is a function of time. In effect, Liu has made the assumption of local similarity[3] by neglecting the time derivative. The two-dimensional (η, t) problem was thus reduced to one in one independent variable (η) where the time (or Damköhler number) enters only as a parameter. It is well known[4,5] that gas-phase diffusion flames in one dimension exhibit multiple (or nonunique) solutions. It is reasonable to expect the same in heterogeneous diffusion flames, as shown in Ref. 1. It is also true that unique solutions occur,[5] but these were overlooked in Ref. 1.

Finally we consider the interpretation of the multiple (or nonunique) solutions. Liu[1] maintains that they correspond to a system stability, allowing the system to stabilize in either a weak or strong combustion mode. This is certainly true in the real one-dimensional case (characterized by a constant Damköhler number). But in the present case, where the Damköhler number is time dependent, we suggest that the system must pass to an unique equilibrium temperature corresponding to an infinite Damköhler number and a strong combustion mode. Physically, the nonunique solutions cannot exist in this case, but appear only as a consequence of the local similarity hypothesis. It is worthwhile to note that this hypothesis breaks down for nonunique solutions because such solutions acquire infinite time-rate of changes. This is contrary to the original hypothesis of slow changes on the time scale inherent in the local similarity hypothesis. Our interpretation of the nonunique solutions is related to the transient behavior of the system, and particularly to that part of the transient behavior which is lost by the local similitude. The nonunique solution corresponds to a very abrupt transition (or ignition) from the kinetic controlled regime (small Damköhler number) to the diffusion controlled regime (large Damköhler number). This is in contrast to the unique type of solution where the transition is a slow one. The author has recently obtained closed form analytic solutions to the problem under discussion.[6] These solutions bear out the above expressed ideas when compared with exact numerical solutions[7,8] for the complete time dependent problem.

One further point is to be made. The problem of fire prevention in space capsules is both physically and mathematically similar to the problems of solid fuel and solid propellant ignition. There already exists a large body of literature (Refs. 2 and 6–10, to cite a few) covering both heterogeneous and gas phase reactions. The problem of Ref. 1 has been specifically treated by Williams.[7] He considered the case of a solid fuel suddenly exposed to an oxidizing atmosphere where both media are initially at the same temperature. Recently, Waldman and Summerfield[8] have extended Williams' theory to cases where the two media are initially at different temperatures. Both studies considered the full set of equations.

In conclusion, an examination of Ref. 1 has revealed serious flaws in formulation, analysis, and conclusions, Moreover, useful solutions to the problem posed in Ref. 1 can be obtained from the existing literature.

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Reply by Author to C. H. Waldman

S. W. Liu*

The Aerospace Corporation, El Segundo, Calif.

In the preceding Comment, a number of questions were raised concerning Ref. 1 of that Comment. In answering, it is first to be pointed out that there can be a difference between the model assumed for solid fuel combustion studies intended for propulsion and that of capsule fire. In Ref. 1, we considered the case where internal heating, due to electrical equipment, raised the temperature of combustible material to a level where appreciable surface reactions occurred. It was assumed that the temperature of a solid was raised instantaneously and uniformly to a given level before the reaction proceeded and remained at that level during combustion. Thus, there is no net local heat transfer in the solid. It was therefore appropriate in our model to direct attention mainly to the gas phase flowfield using the constant surface temperature as a parameter. When such pointwise solutions were used in the discussion of the system stability, they served only to indicate qualitative trends based on the over-all heat and mass balance. This is in contrast to the models assumed in all the references cited by Waldman of solid fuel surface combustion studies orientated to rocket applications in which the only energy source was that due to combustion at the surface and heat was lost primarily by conduction into the relatively cool solid.

The question was also raised by Waldman of the use of the similarity transformation which tacitly invoked the local similarity assumption. It is agreed that something may have been lost, but the discrepancy will not be large except when local true time derivatives of the dependent variables depart appreciably from that depicted by similarity. The materials of concern in a space capsule are by choice all of low flammability. They are therefore a vastly different kind of "fuel" than those considered in the solid fuel combustion for rocket propulsion. Drastic changes of mode in the surface reaction

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* Member of the Technical Staff, Laboratory Operations. Member AIAA.